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AN ANALYSIS OF CABLE AND HOUSING REQUIREMENTS FOR A DEEP-TOWED BODY AT HIGH SPEED

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NOTATION

b	Safety factor
d	Diameter of the cable
F	Drag of the cable per unit length when the cable is parallel to the stream
$k_1, k_2, k_3, k_4, k_5, k_6$	Proportionality factors defined by Equations [13], [14], [16], [18], [19], and [20] respectively
P	See Equation [3]
Q	See Equation [4]
R	Drag of the cable per unit length when the cable is normal to the stream
s	Length of cable
T	Tension in the cable
V	Speed
W	Weight in water per unit length of cable
x, y	Rectangular coordinates of point on cable; distance of towed body aft of tow point and depth of towed body respectively
ξ, η	Nondimensional rectangular coordinates x, y
σ	Nondimensional length of cable s
τ	Nondimensional tension in the cable T
ϕ	Angle of cable to the direction of motion

AN ANALYSIS OF CABLE AND HOUSING REQUIREMENTS FOR A DEEP-TOWED BODY AT HIGH SPEED

by

Leonard Pode

INTRODUCTION

It is frequently desired to tow a body through water at a high speed, keeping it at a fixed position in relation to the towing vessel. If considerable depth is desired, it will usually be desirable to design the body with as large a negative lift-drag ratio as possible in order to overcome the positive lift of the cable. The problem will then be to design such a body and select a cable in such a manner that the body will tow at the desired position with an adequate factor of safety on the breaking strength of the cable.

ANALYSIS

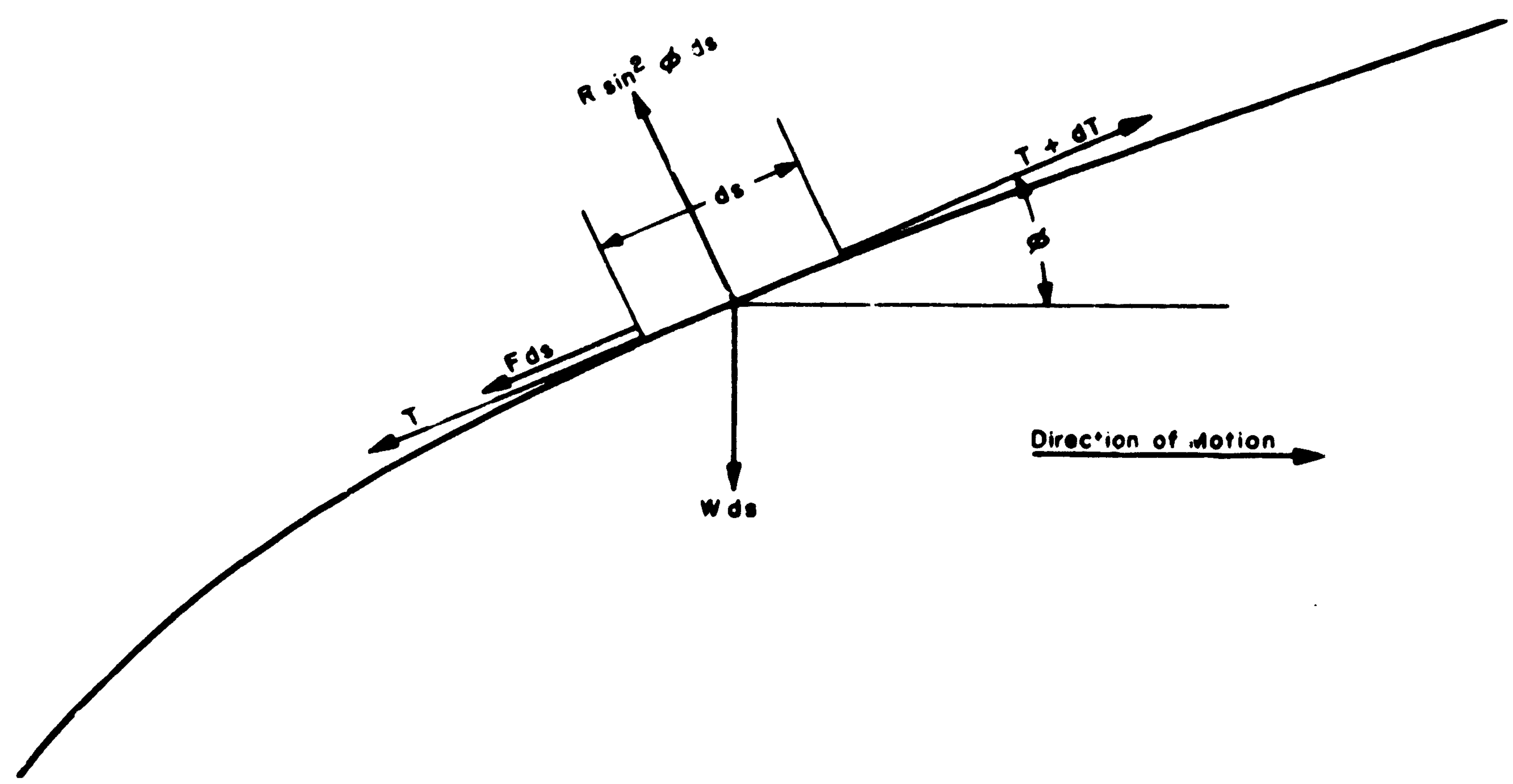

Figure 1 - Forces on an Element of Towing Cable

The physical assumptions will be similar to those explained in reference (1).* The force per unit length parallel to the cable is again assumed to be given by a constant, F; the force per unit length normal to the cable is assumed to be given by $R \sin^2\phi$. In addition to these forces the effect of the weight of the cable will now also be considered. Let T denote the tension,

* Numbers in parentheses indicate references on page 7.

s the length, and ϕ the angle that the element of cable length, ds, makes with the direction of motion. Let W denote the weight per unit length of the cable in water. Assuming that the direction of motion will be perpendicular to that of the gravitational force, the forces may be resolved normal to and along the cable, giving:

$$T \frac{d\phi}{ds} = -R \sin^2 \phi + W \cos \phi \qquad [1]$$

$$\frac{dT}{ds} = F + W \sin \phi \qquad [2]$$

The following definitions are made:

$$P = \frac{F}{R} + \frac{W}{R} \sin \phi \qquad [3]$$

$$Q = \sin^2 \phi - \frac{W}{R} \cos \phi \qquad [4]$$

$$\log_e \tau = \int_{\phi}^{\pi/2} \frac{P}{Q} \, d\phi \qquad [5]$$

$$\sigma = \int_{\phi}^{\pi/2} \frac{\tau}{Q} \, d\phi \qquad [6]$$

$$\xi = \int_{\phi}^{\pi/2} \frac{\tau \cos \phi}{Q} \, d\phi \qquad [7]$$

$$\eta = \int_{\phi}^{\pi/2} \frac{\tau \sin \phi}{Q} \, d\phi \qquad [8]$$

Following a procedure very similar to the demonstration in Reference (1) it can readily be shown that the solution of Equations [1] and [2] leads to the following equations:

$$\frac{T}{T_0} = \frac{\tau}{\tau_0} \qquad [9]$$

$$\frac{Rs}{T_0} = \frac{\sigma - \sigma_0}{\tau_0} \; ; \; \frac{Rs}{T} = \frac{\sigma - \sigma_0}{\tau} \qquad [10]$$

$$\frac{Rx}{T_0} = \frac{\xi - \xi_0}{\tau_0} \quad ; \quad \frac{Rx}{T} = \frac{\xi - \xi_0}{\tau} \qquad [11]$$

$$\frac{Ry}{T_0} = \frac{\eta - \eta_0}{\tau_0} \quad ; \quad \frac{Ry}{T} = \frac{\eta - \eta_0}{\tau} \qquad [12]$$

where the zero subscripts indicate the values of the functions for the point at which the cable joins the body, i.e., where $s = 0$.

If it is desired that a cable of a given type and material be used its weight per unit length, W, is known and is very nearly proportional to the square of its diameter d. Hence

$$W = k_1 d^2 \qquad [13]$$

An approximate value of R based upon experimental data can be obtained from the following formula:

$$R = 1.43 \frac{\rho}{2} V^2 d = k_2 d \qquad [14]$$

where V is the speed and ρ the density of water and $k_2 = 1.43 \frac{\rho}{2} V^2$.

As in Reference (1), the value of F/R is taken to be constant,

$$\frac{F}{R} = 0.022 \qquad [15]$$

In the present problem x and y are given and it will be assumed that the lift-drag ratio of the body, $\tan \phi_0$, is also known, so that ϕ_0 is immediately known.

The symbol b will be used to represent the safety factor of the cable when operating at the given speed. The breaking strength of the cable is very nearly proportional to the square of its diameter, so that the operating tension may be written as

$$T_1 = \frac{k_3 d^2}{b} \qquad [16]$$

It will first be assumed that the weight of the cable can be neglected, i.e., $\frac{W}{R} = 0$. Then the functions τ, σ, ξ, and η become identical with those tabulated in Reference (1) and are functions of ϕ alone.

From Equations [11] and [12]

$$\xi - \xi_0 = \frac{x}{y}(\eta - \eta_0) \qquad [17]$$

This is an equation in functions of ϕ alone and can be solved for ϕ_1, the value of ϕ at the forward end of the cable. Let τ_1, σ_1, ξ_1, and η_1 be the values of the functions corresponding to ϕ_1. With this value of ϕ, Equations [11], [14], and [16] can be used to solve for d as follows:

$$\frac{T_1}{R} = \frac{k_3\, d^2}{b\, k_2\, d} = \frac{\tau_1\, x}{\xi_1 - \xi_0} \;\; ; \;\; d = \frac{k_2\, \tau_1\, x\, b}{k_3(\xi_1 - \xi_0)} = k_4\, b \qquad [18]$$

Substituting this value of d in Equation [16]

$$T_1 = \frac{k_3\, k_4^2\, b^2}{b} = k_5\, b \qquad [19]$$

From Equation [9] T_0 is proportional to T.

$$T_0 = \frac{\tau_0}{\tau_1}\, T_1 = k_6\, b \qquad [20]$$

It is apparent from Equations [18] and [20] that smaller values of b will result in smaller values of both T_0 and d. Since body areas are proportional to T_0, this means that a lower safety factor would make possible the use of a smaller body as well as a smaller cable. The weight of the cable will not materially affect this relationship.

Assuming that it is desired to maintain a given safety factor $b = b_1$, the solution when the weight of the cable can be neglected is immediately available through the following procedure:

1. Solve Equation [17] for $\phi = \phi_1$, using the tables in Reference (1).
2. The diameter d can now be found from Equation [18] using $b = b_1$.
3. Values of R, T, T_0, and s can then be calculated directly.

When the weight of the cable can not be neglected the foregoing method will still give an approximation to d. To obtain a closer approximation the following procedure can be used:

1. The first approximation to d is substituted in Equations [13] and [14] and a value for $\frac{W}{R}$ is obtained.

2. Using this value of $\frac{W}{R}$, tables of P and Q are prepared and new tables for the functions τ, σ, ξ, and η are obtained through numerical integration.

3. Using the new tables for these functions, Equation [17] is again solved for a new value of ϕ.

4. A second approximation to d is then obtained from Equation [18].

Usually the second approximation will be sufficient. Additional accuracy can be obtained by repeating the foregoing process.

EXAMPLE

It is desired to tow a body at 20 knots, at a depth of 1,000 feet and a distance of 750 yards from the towing vessel. The weight of the body is 2500 pounds. The body is equipped with lifting surfaces which give it a lift-drag ratio of tan 70 degrees at this speed. A plow-steel cable will be used and a safety factor of 3 must be maintained.

It is required to find the minimum diameter of the cable, its length, and the necessary lift of the hydrofoil.

The following values are known:

x = 2,250 feet; y = 1,000 feet; ϕ_0 = 70 degrees;

$R = 0.34\,V^2 d$ pounds per foot $= (0.34)\,(20)^2 d$ pounds per foot $= 136d$ pounds per foot; $k_1 = 136 \dfrac{\text{pounds per foot}}{\text{inches}}$

$T = \dfrac{80{,}000\,d^2}{3}$ pounds $= 26{,}667 d^2$ pounds; $k_2 = 80{,}000 \dfrac{\text{pounds}}{\text{inches}^2}$

$W = 1.40\,d^2$ pounds per foot

where d is given in inches and V in knots.

The following values of the cable functions are obtained from the tables of Reference (1) for ϕ_0 = 70 deg:

$\tau_0 = 1.0081$; $\sigma_0 = 0.3655$; $\xi_0 = 0.06452$; $\eta_0 = 0.3577$

Equation [17] can now be solved for $\phi = \phi_1$ as follows:

$$\xi - \xi_0 = \frac{x}{y}(\eta - \eta_0) = \frac{2250}{1000}(\eta - \eta_0)$$

$$2.25\eta - \xi = 2.25\eta_0 - \xi_0 = (2.25)(0.3577) - 0.06452 = 0.74031$$

Values of $2.25\eta - \xi$ are now computed using the tables in Reference (1). At ϕ = 10 degrees and ϕ = 11 degrees, the results are as follows:

ϕ	τ	σ	ξ	η	$2.25\eta - \xi$
10°	1.1343	6.0435	5.1096	2.5458	0.61845
11°	1.1211	5.4495	4.5255	2.4382	0.96045

It is apparent that ϕ_1 must lie between 10 degrees and 11 degrees and can be approximated by interpolating the value $2.25\eta - \xi = 0.74031$. This yields the following results:

$\phi_1 = 10.36$ deg, $\tau_1 = 1.1296$, $\sigma_1 = 5.8319$, $\xi_1 = 4.9015$, $\eta_1 = 2.5075$

The diameter d is now calculated from Equation [18] as follows:

$$d = \frac{k_1 \tau_1 x b}{k_2(\xi_1 - \xi_0)} = \frac{(136)(1.1296)(2250)(3)\ \text{inches}}{(80000)(4.9015 - 0.06452)} = 2.68 \text{ inches}$$

A closer approximation is now obtained. With the value $d = 2.68$ inches a value for $\frac{W}{R}$ is computed as follows:

$$\frac{W}{R} = \frac{1.40d^2}{136d} = \frac{(1.40)(2.68)}{(136)} = 0.02759$$

Using this value of $\frac{W}{R}$, $P = 0.022 + 0.02759 \sin\phi$ and $Q = \sin^2\phi - 0.02759 \cos\phi$.

Tables for P and Q may be prepared and new tables for τ, σ, ξ, and η obtained through numerical integration. Reading the new tables for $\phi_0 =$ 70 deg, $\tau_0 = 1.0181$, $\sigma_0 = 0.3692$, $\xi_0 = 0.06540$, $\eta_0 = 0.3614$.

Equation [17] can now be solved for $\phi = \phi_1$ as above:

$$2.25\eta_1 - \xi = 2.25\eta_0 - \xi_0 = (2.25)(0.3614) - (0.06540) = 0.74775$$

Values of $2.25\eta - \xi$ are computed using the new tables. The following values at $\phi = 12$ deg and $\phi = 13$ deg are obtained.

ϕ	τ	σ	ξ	η	$2.25\eta - \xi$
12°	1.243	7.270	6.243	2.994	0.4935
13°	1.212	6.182	5.180	2.759	1.0277

The following values are then obtained through interpolation:

$$\phi_1 = 12.48°,\ \tau_1 = 1.228,\ \sigma_1 = 6.753,\ \xi_1 = 5.737,\ \eta_1 = 2.882$$

The diameter d is again calculated from Equation [18] as follows;

$$d = \frac{(136)(1.228)(2250)(3)}{(80000)(5.737 - 0.0654)} \text{ inches} = 2.48 \text{ inches}$$

The second approximation will be considered adequate for the purpose. Values of R, T, s, and T_0 can now be calculated directly;

$R = 136d$ pounds per foot $= (136)(2.48)$ pounds per foot $= 337$ pounds per foot

$T_1 = 26{,}667d^2$ pounds $= (26{,}667)(2.48)^2$ pounds $= 164{,}000$ pounds

$$s = \frac{(\sigma_1 - \sigma_0)\,T_1}{\tau_1 R} = \frac{(6.753 - 0.3692)(164{,}000)}{(1.228)(337)} \text{ feet} = 2530 \text{ feet}$$

$$T_0 = \frac{\tau_0}{\tau_1}\,T_1 = \frac{(1.0181)(164{,}000)}{(1.228)} \text{ pounds} = 136{,}000 \text{ pounds}$$

The lift of the body L_0 is calculated as follows:

$$L_0 = T_0 \sin\phi_0 = (136{,}000)(0.93969) \text{ pounds} = 127{,}800 \text{ pounds}$$

Since the weight of the body contributes 2,500 pounds to the lift, the dynamic lift on the body contributes the difference, i.e., 127,800 pounds - 2500 pounds = 125,300 pounds.

REFERENCES

(1) "The Shape and Tension of a Light, Flexible Cable in a Uniform Current," by L. Landweber and M.H. Protter, TMB Report 533, October 1944.